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Integral Representations of (p,q)Forms With Weighted Factors On Complex Manifolds*

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Abstract In this paper we construct a Cauchy-Leray kernel with weighted factors and obtain an integral representation of (p,q)-forms and a Koppelman-Leray-Norguet formula on a complex manifold.

Key words Weighted factor,integral represe ntation,complex manifold.

0 Introduction

In the research of complex functions of several variables,intergral representations hold a very importent position. In 1970s, Henkin obtained the integral representations of the solution of $\bar{\partial}$—equation. After that, integral representations are extensively being studied and anpplied.

In 1982, Andersson & Bendtsson[2] introduced the weighed factors in integral representations in C^n. By choosing appropriate factors, applications of integral representations are more extensive and convinient. In 1990,Andersson [3] obtained the integral representations of (p,q)-forms in a stein mainifold.

In 1989, by using Chern-class theory,Berndtsson [1] obtained the integral representations of (p,q)-froms in a complex manifold. It is the most general integral representations so far.

This paper. as an extension of [1], the author constructs a Cauchy-Leray kernel with weighted factors and obtains an integral representation of (p,q)-forms with weighted factors and Koppelman-Leray-Norguet formula on a complex manifold.

1 Construction of the kernel

Let X be a complex manifold (for example,$M\times M$, M is a complex manifold of complex dimension n). Let Y be a complex submanifold of codimension p (for example,$\triangle$,$\triangle$ is diag-

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nal of $M\times M$). Suppose that $E \xrightarrow{\pi} X$
us a holomorphic vector-bundle of rank p, which has a smooth section η to E and $Y=\{\eta=0\}$. Moreover

$$d\eta_1 \wedge d\eta_2 \cdots \wedge d\eta_p \wedge d\bar{\eta}_1 \wedge d\bar{\eta}_2 \wedge \cdots \wedge d\bar{\eta}_p \neq 0 \text{ on } Y$$

where η_j: s are the coefficients of η about some frame.

Let E^* the dual bundle of E and ξ be an any section of E^*. Let the connections of E and E^* be D and D^*, respectively. Let a local frame of E be $e_1, \cdots e_p$, its duals be $e_1^*, \cdots, e_p^*$. Then

$$e_1^* \wedge e_1 \wedge \cdots \wedge e_p^* \wedge e_p$$

is independent of the choice of frame.

The exterier product on $E^* \oplus E$ can be extended to forms with values in this bundle. In the terms of our local trivialization a form with values in $\wedge^{r,s}$ can be written

$$\alpha = \sum_{|I|=r,|J|=s} a_{IJ} e_I^* \wedge e_J$$

where α_{IJ} are forms, $I=(i_1, \cdots, i_r)$, $J=(j_1, \cdots, j_s)$. If

$$\beta = \Sigma b_{KL} e_K^* \wedge e_L$$

is another such object, ue define:

$$\alpha \wedge \beta = \Sigma a_{IJ} \wedge b_{KL} e_I^* \wedge e_J \wedge e_K^* \wedge e_L$$

In other words, forms can commute with vectors from $E^* \oplus E$ and product between forms is the usual one.

Now, we define the differential d that is independent of the frame. Let

$$d\alpha = \sum_{|I|=r,|J|=s} da_{IJ} e_I^* \wedge e_J$$

Let $\beta=\Sigma b_{KL} e_K^* \wedge e_L$, and a_{IJ} be m-forms, then

$$d(\alpha \wedge \beta) = d\alpha \wedge \beta + (-1)^m \alpha \wedge d\beta$$

Let $\theta=D^2$ be the curvature of E. Let

$$\det(I + \frac{it}{2\pi}\theta) = \sum_{k=0}^{p} C_k[\theta] t^k$$

In terms of Chern-Weil thorem we know that $C_k[\theta]$ is independent of D. Let $\theta=(\theta_{ij})_{p\times p}$ and $\tilde{\theta}=\sum_{i,j=1}^{p} \theta_{ij} e_i^* \wedge e_j$ We have formula

$$(\frac{i}{2\pi})^p \frac{1}{p!} \tilde{\theta}^p = C_p[\theta] e_1^* \wedge e \wedge \cdots e_p^* \wedge e_p$$

It is independent of frame. Let

$$\Lambda = e_1^* \wedge e_1 \wedge \cdots e_p^* \wedge e_p$$

Now, we construct Cauchy-Leray kernel with weighted factors. Let $Q_0, Q_1, \cdots, Q_N$ are smooth sections of E^*. Then $\langle Q_j, \eta \rangle$ is a smooth function in $X\times X$, $j=0,1,\cdots,N$, and

$$d\langle Q_j, \eta \rangle = \langle D^* Q_j, \eta \rangle + \langle Q_j, D\eta \rangle$$

is 1-form in $X\times X$, Let

$$M\Lambda = C_p \sum_{k=0}^{p-1} \binom{p}{k} (-1)^{k-1} \sum_{|\alpha|=p-k} \frac{1}{\alpha!} g^{(\alpha)} (D^* Q_0 \wedge D\eta)^{\alpha_0} \wedge, \cdots \wedge (D^* Q_N \wedge D\eta)^{\alpha_N} \wedge \tilde{\theta}^k$$

where $\alpha_0+\cdots+\alpha_N=p-k,\alpha!=\alpha_0!\cdots\alpha_N!,C_p=(2\pi i)^{-p},g(\lambda_0,\cdots\lambda_N)$ is a holomorphic function, and

$$g^{(\alpha)}=[\frac{\partial^{\alpha_0}}{\partial\lambda_0^{\alpha_0}}\cdots\frac{\partial^{\alpha_N}}{\partial\lambda_N^{\alpha_N}}]g(<Q_0,\eta>,\cdots,<Q_N,\eta>)$$

Let

$$B=-C_p\sum_{|\alpha|=p}\frac{1}{\alpha!}g^{(\alpha)}(D^*Q_0\wedge D\eta)^{\alpha_0}\wedge\cdots\wedge(D^*Q_N\wedge D\eta)^{\alpha_N}$$

Similarly to [3], we have

$$dB=-C_p\sum_{|\alpha|=p}\frac{1}{\alpha!}g^{(\alpha)}d[(D^*Q_0\wedge D\eta)^{\alpha_0}\wedge\cdots\wedge(D^*Q_N\wedge D\eta)^{\alpha_N}]$$

$$=-C_p\sum_{|\alpha|=p}\frac{1}{\alpha!}g^{(\alpha)}\sum_{j=0}^{N}\alpha_jd(D^*Q_j\wedge D\eta)\wedge(D^*Q_0\wedge D\eta)^{\alpha_0}\wedge\cdots$$

$$\wedge(D^*Q_N\wedge D\eta)^{\alpha_j-1}\wedge\cdots\wedge(D^*Q_N\wedge D\eta)^{\alpha_N}$$

Let $N=\mathrm{d}M$ Then we have $N\Lambda=(\mathrm{d}M)\Lambda=\mathrm{d}(M\Lambda)$

Now we replace g in $M\Lambda$ and $N\Lambda$ by $H(\lambda_0)g(\lambda_1,\cdots\lambda_N)$ and Q_0 by $t\xi$. we denote by Mt and Nt the forms so obtained when differentials are also taken with respect to t. we also use the short hand notation

$$(D^*Q\wedge D\eta)^{\alpha}=(D*Q_1\wedge D\eta)^{\alpha_1}\wedge\cdots\wedge(D^*Q_N\wedge D\eta)^{\alpha_N}$$

Thus

$$M_t\Lambda=C_p\sum_{k=0}^{p}\binom{p}{k}(-1)^{k-1}\sum_{j+|\alpha|=p-k}\frac{1}{j!\alpha!}H^{(j)}g^{(\alpha)}(dt\xi\wedge D\eta$$
$$+tD^*\xi\wedge D\eta)^j\wedge(D^*Q\wedge D\eta)^{\alpha}\wedge\theta^k$$

$$N_t\Lambda=-C_p\sum_{j+|\alpha|=p}\frac{1}{j!\ \alpha!}H^{(j)}g^{(\alpha)}d[(dt\xi\wedge D\eta+tD^*\xi\wedge D\eta)^j\wedge(D^*Q\wedge D\eta)^{\alpha}]$$
$$+C_p\sum_{k=1}^{p-1}\binom{p}{k}(-1)^{k-1}\sum_{j+|\alpha|=p-k}\frac{1}{j!\ \alpha!}\{dH^{(j)}\wedge g^{(\alpha)}(dt\xi\wedge D\eta+tD^*\xi\wedge D\eta)^j$$
$$\wedge(D^*Q\wedge D\eta)^{\alpha}\wedge\theta^k$$
$$+H^{(j)}dg^{(\alpha)}\wedge(dt\xi\wedge D\eta+D^*\xi\wedge D\eta)^j\wedge(D^*Q\wedge D\eta)^{\alpha}\wedge\theta^k$$
$$+H^{(j)}g^{(\alpha)}d(dt\xi\wedge D\eta+tD^*\xi\wedge D\eta)^j\wedge(D^*Q\wedge D\eta)^{\alpha}\wedge\theta^k$$
$$+H^{(j)}g^{(\alpha)}(dt\xi\wedge D\eta+tD^*\xi\wedge D\eta)^j\wedge d[(D^*Q\wedge D\eta)^{\alpha}\wedge\theta^k]\}$$

Preposition1. 1 If M_t and N_t are as above, then

$$(d_t+d)M_t=N_t$$

where d is the differential which is independent of t.

Let $H(t)=\exp(-t)$. If $a\neq0$ we define

$$\int_0^{\infty}\exp(-at)t^j\mathrm{d}t=:ji/a^{j+1}$$

This "integral" obeys such rules as differentialiging under the integral sign and formula

$$\int_0^{\infty}\frac{\mathrm{d}}{\mathrm{d}t}x(t)=-x(0)$$

Definition 1. 2 $K(\xi,\eta,Q,g)=\int_0^{\infty}M_t$ is called Cauchy—Lerey Kernal with weighted factors, where the integral on t is d_t in the most right side.

Let $R=\int_0^\infty N_t$ and $P=M_t|_{t=0}$. Thus we have

$$dK=\int_0^\infty dM_t=-\int_0^\infty d_tM_t+\int_0^\infty N_t=M_t|_{t=0}+R=P+R$$

where we let $\langle\xi,\eta\rangle=0$, New we cencretely calculate K, P, R.

The term containing dt in $M_t\Lambda$ is

$$C_p\sum_{k=0}^{p-1}\binom{p}{k}(-1)^k\sum_{j+|\alpha|=p-k}\frac{1}{j!\alpha!}H^{(j)}g^{(\alpha)}\xi\wedge D\eta(D^*\xi\wedge D\eta)^{j-1}\wedge(D^*Q\wedge D\eta)^\alpha\wedge\bar\partial^k\wedge jt^{j-1}dt$$

Therefore,

$$K\Lambda=\int_0^\infty\mu_t\Lambda=C_p\sum_{k=0}^{p-1}\binom{p}{k}(-1)^k\sum_{j+|\alpha|=p-k}\frac{1}{\alpha!}g^{(\alpha)}\frac{\xi\wedge D\eta(D^*\xi\wedge D\eta)^{j-1}}{<\xi,\eta>^j}\wedge(D^*Q\wedge D\eta)^\alpha\wedge\bar\partial^k \tag{1.3}$$

Because $\xi\wedge\xi=0$, so when we replace ξ in (1,3) by $\varphi\xi$ (1,3) is invariant.

$$P\Lambda=M_t|_{t=0}\Lambda=M_t\Lambda|_{t=0} \tag{1.4}$$

$$=C_p\sum_{k=0}^{p-1}\binom{p}{k}(-1)^k\sum_{|\alpha|=p-k}\frac{1}{\alpha!}g^{(\alpha)}(D^*\xi\wedge D\eta)^\alpha\wedge\bar\partial^k$$ Let $\xi=\Sigma\xi_je_j^*$, $\eta=\Sigma\eta_je_j$, $D^*\xi=\Sigma(D^*\xi)_je_j^*$, $D\eta=\Sigma(D\eta)_je_j$,

$$C^*\xi=d(D^*\xi)=\Sigma d(D^*\xi)e_j^* \text{ and } c\eta=d(D\eta)=\Sigma d(D\eta)_je_j.$$

Then the first term of { } in Nt which contains dt is

$$-H^{(j+1)}g^{(\alpha)}[<\xi,\eta>(D^*\xi\wedge D\eta)^j-j(<D^*\xi,\eta>+<\xi,D\eta>)\wedge\xi\wedge D\eta$$
$$\wedge(D^*\xi\wedge D\eta)^{j-1}]\wedge(D^*Q\wedge D\eta)^\alpha\wedge\bar\partial^k\wedge t^jdt$$

where

$$dH^{(j)}=H^{(j+1)}d(-t<\xi,\eta>)$$
$$=-H^{(j+1)}[dt<\xi,\eta>+t(<D^*\xi,\eta>+<\xi,D\eta>)]$$

The second term of { } in Nt Λ which contains dt is

$$-H^{(j)}\sum_{l=1}^N\frac{\partial}{\partial\lambda_l}g^{(\alpha)}(<D^*Q_l,\eta>+<Q_l,D\eta>)\wedge\xi\wedge D\eta$$
$$\wedge(D^*\xi\wedge D\eta)^{j-1}\wedge(D^*Q\wedge D\eta)^\alpha\wedge\bar\partial^k\wedge t^{j-1}dt$$

The third tern of {} in Nt which contains dt is

$$H^{(j)}g^{(\alpha)}[(D^*\xi\wedge D\eta)^j-(d\xi\wedge D\eta+\xi\wedge c\eta)\wedge(D^*\xi\wedge D\eta)^{j-1}$$
$$+\xi\wedge D\eta\wedge(j-1)(C^*\xi\wedge D\eta-D^*\xi\wedge c\eta)\wedge(D^*\xi\wedge D\eta)^{j-2}]$$
$$\wedge(D^*Q\wedge D\eta)^\alpha\wedge\bar\partial^k\wedge jt^{j-1}dt$$

The fourth term of { } in Nt which contains dt is

$$H^{(j)}g^{(\alpha)}\xi\wedge D\eta\wedge(D^*\xi\wedge D\eta)^{j-1}\wedge[\sum_{l=1}^N\alpha_l(c^*Q_l\wedge D\eta-D^*Q_l\wedge C\eta)$$
$$\wedge(DQ\wedge D\eta)^{\alpha_1\cdots\alpha_l-1\cdots\alpha_N}\wedge\bar\partial^k+(D^*Q\wedge D\eta)^\alpha\wedge\bar\partial^{k-1}]$$

where

$$(DQ\wedge D\eta)^{\alpha_1\cdots\alpha_l-1\cdots\alpha_N}=(D^*Q_1\wedge D\eta)^{\alpha_1}\cdots\wedge(D^*Q_l\wedge D\eta)^{\alpha_l-1}\cdots\wedge(D^*Q_N\wedge D\eta)^{\alpha_N}$$

Therefore we have

$$RA = \int_0^\infty N_t \tag{1.5}$$

$$= C_p \sum_{j+|\alpha|=p} \frac{1}{\alpha!} g^{(\alpha)} \Big[\frac{d\xi \wedge D\eta \wedge (D\xi^{*} \wedge D\eta)^{j-1} \wedge (D^{*}Q \wedge D\eta)^{\alpha}}{<\xi,\eta>^{j}}$$

$$+ \frac{\xi \wedge C\eta \wedge (D\xi^{*} \wedge D\eta)^{j-1} \wedge (D^{*}Q \wedge D\eta)^{\alpha}}{<\xi,\eta>^{j}}$$

$$- \frac{\xi \wedge D\eta \wedge (j-1)(C^{*}\xi \wedge D\eta - D^{*}\xi \wedge c\eta) \wedge (D^{*}\xi \wedge D\eta)^{j-2} \wedge (D^{*}Q \wedge D\eta)^{\alpha}}{<\xi,\eta>^{j}}$$

$$- \frac{\xi \wedge D\eta \wedge (D\xi * \wedge D\eta)^{j-1}}{<\xi,\eta>^{j}}$$

$$\cdot \sum_{l=1}^{N} \alpha_l (C^{*}Q_l \wedge D\eta - D^{*}Q_l \wedge C\eta)(D^{*}Q \wedge D\eta)^{\alpha_1 \cdots \alpha_l - 1 \cdots \alpha_l} \Big]$$

$$- j \frac{(<D^{*}\xi,\eta> + <\xi,D\eta>) \wedge \xi \wedge D\eta \wedge (D^{*}\xi \wedge D\eta)^{j-1} \wedge (D^{*}Q \wedge D\eta)^{\alpha}}{<\xi,\eta>^{j+1}} \Big]$$

$$+ C_p \sum_{k=1}^{p-1} \binom{p}{k} (-1)^{k-1} \sum_{j+|\alpha|=p-k} \frac{1}{j!\alpha!}$$

$$\cdot \{ g^{(\alpha)}$$

$$\frac{j(<D^{*}\xi,D\eta> + <\xi,D\eta>) \wedge \xi \wedge D\eta \wedge (D^{*}\xi \wedge D\eta)^{j-1} - <\xi,\eta> (D^{*}\xi \wedge D\eta)^{j-1}}{<\xi,\eta>^{j}}$$

$$\wedge (D^{*}Q \wedge D\eta)^{\alpha} \wedge \theta^{k}$$

$$- \sum_{l=1}^{N} \frac{\partial}{\partial \lambda_l} g^{(\alpha)} (<D_l^{*}, \wedge D\eta> + <Q_l, D\eta>) \wedge \frac{\xi \wedge D\eta \wedge (D^{*}\xi \wedge D\eta)^{j-1}}{<\xi,\eta>^{j}} \wedge (D^{*}Q \wedge D\eta)^{\alpha} \wedge \theta^{k}$$

$$+ g^{(\alpha)} \Big[\frac{D^{*}\xi \wedge D\eta)^{j} - (d\xi \wedge D\eta + \xi \wedge (\eta) \wedge (D^{*}\xi \wedge D\eta)^{j-1}}{<\xi,\eta>^{j}}$$

$$+ \frac{(j-1)\xi \wedge D\eta \wedge (C^{*}\xi \wedge D\eta - D\xi \wedge C\eta) \wedge (D^{*}\xi \wedge D\eta)^{j-2}}{<\xi,\eta>^{j}} \Big] \wedge (D^{*}Q \wedge D\eta)^{\alpha} \wedge \theta^{k}$$

$$+ g^{(\alpha)} \frac{\xi \wedge D\eta \wedge (D^{*}\xi \wedge D\eta)^{j-1}}{<\xi,\eta>^{j}} \wedge \Big[\sum_{l=1}^{N} \alpha_l (C^{*}Q_l \wedge D\eta - D^{*}Q_l \wedge C\eta)$$

$$\wedge (D^{*}Q \wedge D\eta)^{\alpha_1 \cdots \alpha_l - 1 \cdots \alpha_N} \wedge \theta^{k}$$

$$+ (D^{*}Q \wedge D\eta)^{\alpha} \wedge k d\theta \wedge \theta^{k-1} \Big] \}$$

Espacially, if $g \equiv 1, Q_l \equiv 0, l = 1, \cdots N$, we get

$$KA = C_p \sum_{k=0}^{p-1} \binom{p}{k} (-1)^{k} \frac{\xi \wedge D\eta \wedge (D^{*}\xi, \wedge D\eta)^{p-k-1}}{<\xi,\eta>^{p-k}} \wedge \theta^{k} \tag{1.6}$$

This is the Cauchy-Leray kernal constructed by B. Berndtsson in [1]. And

$$PA \equiv 0, \quad \text{Thus} \quad P \equiv 0 \tag{1.7}$$

$$RA = C_p \sum_{k=0}^{p-1} \binom{p}{k} (-1)^{k} \Big[\frac{(d\xi \wedge D\eta \wedge + \xi \wedge C\eta) \wedge (D^{*}\xi \wedge D\eta>^{p-k-1}}{<\xi,\eta>^{p-k}} \wedge \theta^{k} \tag{1.8}$$

$$- \frac{(p-k-1)\xi \wedge D\eta \wedge (C^{*}\xi \wedge D\eta - D^{*}\xi \wedge C\eta) \wedge (D^{*}\xi \wedge D\eta)^{p-k-2}}{<\xi,\eta>^{p-k}} \wedge \theta^{k}$$

$$- \frac{(p-k)(<D^{*}\xi,\eta> + <\xi,D\eta> \wedge \xi \wedge D\eta) \wedge (D^{*}\xi \wedge D\eta)^{p-k-1}}{<\xi,\eta>^{p-k-1}} \wedge \theta^{k}$$

$$+ \frac{\xi \wedge D\eta \wedge (D^{*}\xi \wedge D\eta)^{p-k-1}}{<\xi,\eta>^{p-k}} \wedge k d\theta \wedge \theta^{k-1}$$

2 Integral representation of (p,q) -forms with weighted factors

Now let M be a complex manifold and its complex dimension be n. Let $X=M\times M$ and E be a holomorphic vector bundle on X whose rank is n. Let η be a section of E so that

$$\triangle = \{\eta = 0\} = \{(\xi,z) \in M \times M, \xi = z\}$$

Definition 2.1 Let ξ be a smooth section of E^*. ξ is called allowable about η if for any A $\subset\subset$X we have

$$|\xi| \leqslant C_A|\eta|, \qquad |<\xi,\eta>1 \geqslant C_A|\eta|^2$$

where C_A and C_A are constants only related with A.

Let $g\equiv 1$. If ξ is alloable near $\triangle$ and K,P, and R are (1.6) (1.7) (1.8), repectively, then

$$dK = [\triangle] + R \qquad (R = - C_n[\theta]) \tag{2.2}$$

where $[\triangle]$ denotes the current that integrations is over $\triangle\subset M\times M$. This is the result of [1]. If K, P and R are general, i.e. as in (1.3) (1.4) and (1.5), noting that the principal part of K is just as in (1.6) we get from (2.2) following theorem:

Theorem 2.3 If ξ is allowable about and $g(0)=1$, following formula is satified

$$dK = [\triangle] + R + P$$

Theorem 2.4 If $D\subset\subset M$ is a domain with $C^{(1)}$ — boundary and if f is any smooth k — form, we have the formula

$$f(z) = \int_D K \wedge \mathrm{d}f + d\int_D K \wedge f + (-1)^k\int_{\partial D} K \wedge \int + \int_D P \wedge f + \int_D R \wedge f, z \in D$$

If E is a holomorphic vetor bondle and η is its section, then for any $C^{(1)}(p,q)$ — form f(z) in D we have

$$f(z) = \int_D K_{p,q} \wedge \bar{\partial} f + \bar{\partial}\int_D K_{p-q-1} \wedge f + (-1)^{p+q}\int_{\partial D} K_{p,q} \wedge f$$
$$+ \int_D P_{p,q} \wedge f + \int_D R_{p,q} \wedge f$$

where lower index (p,q) means the component which has bidegree (p,q) in z and $K_{p,-1}\equiv 0$

The proof of the theorem is standard and we omit it.

3 The Koppelman-Leray-Norguet formula with weighted factors

Definition 3.1 Let M be a complex manifold of complex dimension n. An open set $D\subset\subset M$ is called to have a piecewise $C^{(1)}$-boundary if there exists a finite open covering $\{U_j\}_{j=1}^{h}$ of an open neibourhood U of ∂D and $C^{(1)}$-functions $\rho_j \to (1\leqslant j\leqslant h)$ such that

1) $D \wedge U = \{z \in U =$ for $1 \leqslant j \leqslant h$, either $z \overline{\in} U_j$ or $\rho_j(z) < 0\}$

2) for $1 \leqslant i_1 < \cdots < i_l \leqslant h$, the 1-forms $d\rho_{i_1}, \cdots, d\rho_{i_l}$

are linearly independent over R at every point of $\bigcap_{v=1}^{l} U_{iv}$. And $\{U_j,\rho_i\}$ is called a frame of D.

For every ordered subset $I=\{i_1,\cdots,i_l\}$ of $\{1,2,\cdots,h\}$ define

$$S_I = \{z \in \partial D \cap (\bigcap_{i\in I} U_i): \rho_i(x) = 0, i \in I\}$$

and choose the orientation on S_I such that the orientation is skew symmetric in the component of I and follwing two equation hold when D is given the natural orientation:

$$\partial D = \sum_{j=1}^{h} S_j, \partial S_I = \sum_{j=1}^{h} S_{Ij}$$

Let

$$\sigma = \{\lambda = (\lambda_0,\cdots,\lambda_h) \in R^{h+1}, \lambda_j \geqslant 0, \sum_{j=0}^{h}\lambda_j = 1\}$$

be the standard h-simplex in R^{h+1} with cononical orientation, for every ordered subset $J=\{j_1,\cdots,j_m\}$ of $\{0,\cdots,h\}$ with strictly increasing components, set

$$\sigma_J = \{\lambda \in \sigma: \sum_{j\in J}\lambda_j = 1\}$$

The orientation of each σ_J is chosen so that

$$\partial\sigma_J = \sum_{v=1}^{m}(-1)^{v-1}\sigma_{j_1\cdots\hat{j}_v\cdots j_m}$$

where $\hat{j}_v$ means that j_v is omitted. With this orientation we have

$$\partial(S_I \times \sigma_J) = \partial S_I \times \sigma_J + (-1)^{|I|} S_I \times \partial_J$$

Lemma 3. 2

$$\partial(\sum_I(-1)^{|I|}S_I \times \sigma_{0I}) = \sum_I S_I \times \sigma_I - \partial D \times \sigma_0$$

where the summation is over all ordered subsets I of $\{1,\cdots,h\}$ with strictly increasing components, and $0I=\{0,i_1,\cdots,i_l\}$

Let $\xi,\xi_1,\cdots\xi_h$ be sections of E^* which are allowable about η. Let g and Q_l be as in ξi. Let

$$t^*(\xi,\xi_1,\cdots,\xi_h,\eta,\lambda) = \sum_{l=1}^{h}\lambda_l\frac{\xi_l}{<\xi_l,\eta>} + \lambda_0\frac{\xi}{<\xi,\eta>}$$

and

$$\overline{K}(\xi,\xi_1,\cdots,\xi_R,\eta,Q,g,\lambda)\Lambda$$

$$= C_n\sum_{k=0}^{n-1}\binom{n}{k}(-1)^k\sum_{j+|\alpha|=n-k}\frac{1}{\alpha!}g^{(\alpha)}t^* \wedge D\eta \wedge (D^*t^* \wedge D\eta)^{j-1} \wedge (D^*Q \wedge D\eta)^\alpha \wedge \bar{\theta}^k$$

Lemma 3. 3 $\overline{K}(\xi,\xi_1,\cdots\xi_h,\eta,Q,g\lambda)|\sigma_{0I} = \overline{K}(\xi_1,\cdots\xi_h,\eta,Q,g,\lambda)$

$\overline{K}(\xi,\xi_1,\cdots\xi_h,\eta,Q,g\lambda)|\sigma_0 = \overline{K}(\xi,\eta,Q.g)$

Lemma 3. 4 $d\overline{K}(\xi,\xi_1,\cdots,\xi_h,\eta,Q,g,\lambda)$

$$= (d_{\xi,z} + d_\lambda)\overline{K}(\xi,\xi_1,\cdots,\xi_h,\eta,Q,g,\lambda)$$

$$= C_n\sum_{j+|\alpha|=n}\frac{1}{\alpha!}g^{(\alpha)}[dt^* \wedge D\eta \wedge (D^*t^* \wedge D\eta)^{j-1} \wedge (D^*Q \wedge D\eta)^\alpha$$

$$+ t^* \wedge C\eta \wedge (D^*t^* \wedge D\eta)^{j-1} \wedge (D^*Q \wedge D\eta)^\alpha$$

$$- t^* \wedge D\eta \wedge (j-1)(C^*t^* \wedge D\eta - D^*t^* \wedge C\eta) \wedge (D^*t^* \wedge D\eta)^{j-2}$$

$$\wedge (D^*Q \wedge D\eta)^\alpha$$

$$- t^* \wedge D\eta \wedge (D^*t^* \wedge D\eta)^{j-1} \wedge \sum_{l=1}^{N}\alpha_l(C^*Q_l \wedge D\eta - D^*Q_l \wedge C\eta)$$

$$
\begin{aligned}
&\wedge (D^{*}Q \wedge D\eta)^{\alpha_1\cdots\alpha_l-1\cdots\alpha_N}]\\
&+ C_n\sum_{k=1}^{n-1}\binom{n}{k}(-1)^k \sum_{j+|\alpha|=n-k}\frac{1}{\alpha!}\{\sum_{l=1}^{N}\frac{\partial}{\partial\lambda_l}g^{(\alpha)}(<D^{*}Q_l,\eta>+<Q_l,D\eta>)\\
&\quad\wedge t^{*}\wedge D\eta\wedge(D^{*}t^{*}\wedge D\eta)^{j-1}\wedge(D^{*}Q\wedge D\eta)^{\alpha}\wedge\bar\theta^{k}\\
&+ g^{(\alpha)}dt^{*}\wedge D\eta\wedge(D^{*}t^{*}\wedge D\eta)^{j-1}\wedge(D^{*}Q\wedge D\eta)^{\alpha}\wedge\bar\theta^{k}\\
&+ g^{(\alpha)}t^{*}\wedge C\eta\wedge(D^{*}t^{*}\wedge D\eta)^{j-1}\wedge(D^{*}Q\wedge D\eta)^{\alpha}\wedge\bar\theta^{k}\\
&- g^{(\alpha)}t^{*}\wedge D\eta\wedge j-1(C^{*}t^{*}\wedge D\eta-D^{*}t^{*}\wedge C\eta)\wedge(D^{*}t^{*}\wedge D\eta)^{j-2}\\
&\wedge(D^{*}Q\wedge D\eta)^{\alpha}\wedge\bar\theta^{k}\\
&- g^{(\alpha)}t^{*}\wedge D\eta\wedge(D^{*}t^{*}\wedge D\eta)^{j-1}\wedge\sum_{l=1}^{N}\alpha_l(C^{*}Q_l\wedge D\eta-D^{*}Q_l\wedge C\eta)\\
&\wedge(D^{*}Q\wedge D\eta)^{\alpha_1\cdots\alpha-1\cdots\alpha_N}\wedge\bar\theta^{k}\\
&- g^{(\alpha)}t^{*}\wedge D\eta\wedge(D^{*}t^{*}\wedge D\eta)^{j-1}\wedge(D^{*}Q\wedge D\eta)^{\alpha}\wedge kd\bar\theta\wedge\bar\theta^{k-1}\\
&\triangleq T(\xi,\xi_1,\cdots,\xi_h,\eta,Q,g,\lambda)\Lambda
\end{aligned}
$$

The proofs of Lemma 3.3 and Lemma 3.4 are simple and are ommtted.

Theorem3.5 Let $D\subset\subset M$ and its boundery∂D be $C^{(1)}$—piecewise. Let $\xi,\xi_1,\cdots,\xi_h$ be sections of E^{*} that are allowable about η. If f is a (p,q) —form whose coefficients are in $C^{(1)}(D)$, then we have following formula:

$$
\begin{aligned}
f(z) &= (-1)^{p+q}\sum_I\int_{S_I\times\sigma_I}\bar K_{p,q}\wedge f+(-1)^{p+q+1}\sum_I(-1)^{(I)}\int_{S_I\times\sigma_{OI}}\bar T_{p,q}\wedge f\\
&\quad-\sum_I(-1)^{|I|}\int_{S_I\times\sigma_{OI}}\bar K_{p,q}\wedge\bar\partial f+\int_D\bar K_{p,q}\wedge\bar\partial f+\bar\partial_z\int_D\bar K_{p,q-1}\wedge f\\
&\quad+\int_D R_{p,q}\wedge f+\int_D P_{p,q}\wedge f,\quad z\in D
\end{aligned}
$$

The proof of Theorem 3.5 Applying Theorem 2.4 and Stokes formula, we have

$$
\begin{aligned}
\int_{\partial D\times\sigma_0}\bar K\wedge f &= \sum_I\int_{S_I\times\sigma_I}\bar K\wedge f-\sum_I(-1)^{|I|}\int_{S_I\times\sigma_{OI}}d(\bar K\wedge f)\\
&=\sum_I\int_{S_I\times\sigma_I}\bar K\wedge f-\sum_I(-1)^{|I|}\int_{S_I\times\sigma_{OI}}\bar T\wedge f\\
&\quad+(-1)^{p+q-1}\sum_I(-1)^{|I|}\int_{S_I\times\sigma_{QI}}\bar K\wedge\bar\partial f
\end{aligned}
$$

Therefore the theorem is proved.

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On The Second Order Neutral Delay Difference Equations with Variable Coefficients

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Abstract In this paper, we are concerned with the oscillation and nonoscillation of the second order neutral delay difference equation

$$\Delta^2(X_n - C_nX_{n-m}) = p_nX_{n-k}, n \geqslant n_0 \qquad (*)$$

where, C_n, p_n are real numbers, m, k, n_0 are given nonnegative integers such that $p_n \geqslant 0$, $p_n \not\equiv 0$, $n \geqslant n_0$ and $m \geqslant 1$. $\Delta Xn = X_{n+1} - Xn$. We show that if $C_n \geqslant 0$ then E_q. (*) has a unbounded positive solution. Sufficient and also necessary and sufficient conditions are obtained for oscillation of all bounded solutions of E_q. (*).

Key Word Neutral difference equation, oscillation, nonoscillation

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复流形上(p,q)形式的带权因子的积分表示

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摘　要 本文在[1]的基础上，通过构造带权的 Cauchy-Leray 核，得到了一般复流形上的(p,q)形式的带权因子的积分表示和带权子的 Koppelman-Lerey-Noryuet 公式.

关键词 权因子，积分表示，复流形